Uncertainty in Energy Systems

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Outline

1 Introduction
- Uncertainty in Energy
- Handling Uncertainty

2 Ingredients
- Probust Constraints
- With Recourse

3 Perspectives
- Exploit Block Structure

Uncertainty

- In energy management problems, one seeks to manage resources in some cost optimal way.
- These resources can involve generation levels of given assets (e.g., power plants), demand side management tools or other levers such as voltage levels in (distribution) substations...
- Uncertainty in energy impacts not only the objective function but also the constraint (structure)

Uncertainty II

- In several (short term) energy management optimization problems the decisions have to be taken prior to observing any uncertainty.
- Some decisions may naturally lack the possibility for recourse if one thinks of “commitment choices” in unit commitment (long start up delays, minimum up/down times etc...), changing voltage levels in substations or decisions involving manual actions (e.g., changes in topology in DSO networks), or again investment. Others, e.g., power levels may allow adjustment to some extent.
- Accounting for uncertainty has to be cast into a proper mathematical framework.

Uncertainty III

Key uncertainty factors that impact the system are

- (Weather related): Wind / Solar generation, hydro inflows
- (Weather dependent): Customer load
- (System related): (partial) outages, prices of various commodities

But the base weather pattern is also potentially dependent on meta-uncertainty: climate change.

Formulating Problems

Answers to the following questions fully shape the formulated model

- Do we need to account for recourse and if so, to what extend: 1-stage, 2-stage or multi-stage models?
- If uncertainty impacts constraints in some “unhedgeable" way, how are these constraints given a meaning: in probability, robust, probust?
- Are the costs in fact a distribution? What risk functional transforms this into a “number"? : expectation, CVaR, worst-case criteria?

Link to Statistics (Data science)

The approaches that can deployed are not entirely independent from what can be reasonably known about uncertainty itself:

- Can one “know" the distribution of a given uncertainty factor (clearly yes if data is available: load, inflows, weather)
- Do we only “know" some loose bounds, some area in which lives a given factor ?
- Do we only have finitely many samples?
- Do we have “Atmospheres", i.e., meta-scenarios? : a set of assumptions to test in “what-if-mode", impossible to reasonably assign a probability too?

SP answer to meta-uncertainty

What if we are uncertain about our characterization of uncertainty? In stochastic programming, two pathways provide an explicit answer to this question:

- Stability: the study of perturbation of a given model, when “our characterization” of uncertainty is perturbed
- Ambiguity / Distributional robustness: Rather than requesting certain “measurements" to hold for a given probability measure, a worst case is considered over a “special" family

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Probust / Generalized probability constraint

- Let us consider the following general setting:

$$\varphi(x) := \mathbb{P}[g_t(x, \xi) \leq 0, \ \forall t \in T(x)],$$

where $\varphi : X \to [0, 1]$ is the / a probability function,

- typically involved in a probabilistic restriction of the form:

$$\varphi(x) \geq p,$$

with p some user specified safety-value.

Recent Insights

- Significant progress of late in the understanding of differentiability of probability constraints, e.g., [van Ackooij et al.(2019b)](and references therein)
- Significant progress in the understanding of convexity (of the feasible set), e.g., [van Ackooij and Malick(2019)], but also [Minoux and Zorgati(2016), van Ackooij(2015)].
- Faster, better algorithms, e.g., [Bremer et al.(2015)], [van Ackooij and de Oliveira(2014)]: Problems involving "joint" probability constraints with random vectors of dimension approx. 200 can now be solved within a "couple of minutes".

Several decision stages

- We pick the paradigm that a first decision $x_1 \in \mathbb{R}^{n_1}$ needs to be taken, uncertainty ξ is revealed and then a second “recourse” decision has to be taken $x_2 \in \mathbb{R}^{n_2}$ (etc...)
- For simplicity of presentation, the underlying model structure is assumed linear.

Two decision stages

- The second (stage) decision acts on (x_1, ξ) and solves

$$\begin{aligned}\min_{x_2} \quad & q(\xi)^{\mathsf{T}} x_2 \\ \text{s.t.}\, & T(\xi)x_1 + W(\xi)x_2 \leq h(\xi) \\ & x_2 \in X_2,\end{aligned}$$

- Let the optimal cost be denoted by $c(x_1, \xi)$.
- Then the optimization problem to be solved is

$$\begin{aligned}\min_{x_1} \quad & c^{\mathsf{T}} x_1 + \mathbb{E}\left(c(x_1, \xi)\right) \\ \text{s.t.}\, & Ax_1 \leq b \\ & x_1 \in X_1,\end{aligned}$$

Extensions

- The underlying linear structure is not mandatory
- with several decision stages we get:

$$\min_{\substack{x_1 \in X_1 \\ A_1 x_1 = b_1}} c_1^\top x_1 + \mathbb{E}_{|\xi_1}\left[\min_{\substack{x_2 \in X_2 \\ B_2 x_1 + A_2 x_2 = b_2}} c_2^\top x_2 + \mathbb{E}_{|\xi_{[2]}}\left[\cdots + \mathbb{E}_{|\xi_{[T-1]}}[\min_{\substack{x_T \in X_T \\ B_T x_{T-1} + A_T x_T = b_T}} c_T^\top x_T] \right] \right], \tag{1}$$

where some of (or all) data $\xi = (c_t, B_t, A_t, b_t)$ can be subject to uncertainty for $t = 2, \ldots, T$. The expected value $\mathbb{E}_{|\xi_{[t]}}[\,\cdot\,]$ is taken with respect to the conditional probability measure of the random vector $\xi_t \in \Xi_t$, defining the stochastic process $\{\xi_t\}_{t=1}^T$. Furthermore, $X_t \neq \emptyset$, $t = 1, \ldots, T$, are polyhedral convex sets that do not depend on the random parameters, which we denote by $X_t := \{x_t \in \mathbb{R}_+^{n_t} \mid D_t x_t = d_t\}$.

Recent Insights

- Two stage models of now very large size can be solved:
  - various layers of decomposition (Benders), e.g., [van Ackooij and Malick(2016)]
  - Stabilization (bundle-like approaches) and inexact information requirements at certain iterates: [de Oliveira et al.(2011)]
  - Dynamic clustering of uncertainty, e.g., [van Ackooij et al.(2018)]

- Multi-stage models are very challenging and best solved by SDDP or Dynamic programming based approaches. Stabilization is not fully understood yet, but some studies are in progress e.g., [van Ackooij et al.(2019a)][Asamov and Powell(2018)]

Blocks

- Since modelling an energy system requires incorporating a variety of the previously discussed features, it is clear that a “monolithic" model and approach will not allow for efficient handling.
- This is the angle of attack chosen in plan4res: plan4res
- Indeed, energy models are block-structured: exploiting this structure is what will yield efficient algorithms (and thus understandable results, policies, recommendations and studies).
- Exploiting block-structure will also allow us to co-use various methods and modelling features together

Computational effort a 2stage algorithm for UC

The global computational effort is indeed "small", when one recalls that this corresponds to solving 510 full large-scale UC problems.

Table: Results with $\delta = 1\%$

Instance	Primal Recovery Approach	Nb. Iter	Oracle Calls 1st Stage	Oracle Calls 2nd Stage	Gaps (%) Δ_G^*	Gaps (%) Δ_A^*	$f(x^*)$ ($\times 10^7$)
L1	LagHeur.TD	8	361	1741	1.48	0.66	5.6459
L1	LagHeur.TI	8	417	1496	0.27	0.5486	5.5410
L1	LagHeur.TB	6	267	1043	0.633	0.8716	5.58382
L2	LagHeur.TD	8	404	2441	1.593	0.738	5.64828
L2	LagHeur.TI	9	445	2238	0.312	0.915	5.55953
L2	LagHeur.TB	11	458	2607	1.589	0.406	5.60803
L3	LagHeur.TD	11	516	3817	1.6651	0.711	5.64805
L3	LagHeur.TI	6	293	1553	0.198	0.7122	5.54794
L3	LagHeur.TB	8	317	2374	0.679	0.628	5.58464

Summary

In this talk we have discussed various aspects of handling uncertainty in energy models: theoretical, algorithmic and practical aspects were discussed.

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